EIE312 COMMUNICATIONS PRINCIPLES

Outline:

Principles of communications:

1. An elementary account of the types of transmission (Analogue signal transmission and digital signal transmission). Block diagram of a communication system.
2. Brief Historical development on communications:
    a. Telegraph
    b. Telephony
    c. Radio
    d. Satellite
    e. Data
    f. Optical and mobile communications
    g. Facsimile
3. The frequency Spectrum
4. Signals and vectors, orthogonal functions.
5. Fourier series, Fourier integral, signal spectrum, convolution, power and energy correlation.
6. Modulation, reasons for modulation, types of modulation.
7. Amplitude modulation systems:
    a. Comparison of amplitude modulation systems.
    b. Methods of generating and detecting AM, DBS and SSB signals.
    c. Vestigial sideband
    d. Frequency mixing and multiplexing, frequency division multiplexing
    e. Applications of AM systems.
8. Frequency modulation systems:

a. Instantaneous frequency, frequency deviation, modulation index, Bessel coefficients, significant sideband criteria

b. Bandwidth of a sinusoidally modulated FM signal, power of an FM signal, direct and indirect FM generation,

c. Various methods of FM demodulation, discriminator, phase-lock loop, limiter, pre-emphasis and de-emphasis, Stereophonic FM broadcasting

9. Noise waveforms and characteristics. Thermal noise, shot noise, noise figure and noise temperature. Cascade network, experimental determination of noise figure. Effects of noise on AM and FM systems.

10. Block diagram of a superheterodyne AM radio receiver, AM broadcast mixer, local oscillator design, intermodulation interference, adjacent channel interference, ganging, tracking error, intermediate frequency, automatic gain control (AGC), delay AGC, diode detector, volume control.

11. FM broadcast band and specification, Image frequency, block diagram of a FM radio receiver, limiter and ratio detectors, automatic frequency control, squelch circuit, FM mono and FM stereo receivers.

12. AM broadcast band and specification.

13. TV broadcast band and specification. Signal format, transmitter and receiver block diagrams of black and white TV and colour TV.

MODULATION

3.1. Introduction

Different types of signals that are generally encountered in communication systems were discussed in the first chapter. Many of these signals have frequency spectra that is not suitable for direct transmission especially when atmosphere is used as the transmission channel. In such a case, the frequency spectra of the signal may be translated by modulating high frequency carrier wave with the signal. Consider, for example, picture signal of a TV camera. It has a frequency spectra of DC to 5.5 MHz. Such a wide band of frequencies cannot be propagated through ionosphere. However, if this signal is modulated with a carrier in VHF or UHF range, the percentage bandwidth becomes very small and the signal becomes suitable for transmission through atmosphere.

Apart from this primary requirement for modulation of signals, there are additional objectives which are met by modulation.

(*a*) *Ease of radiation.* As the signals are translated to higher frequencies, it becomes relatively easier to design amplifier circuits as well as antennae systems at these increased frequencies.

(*b*) *Adjustment of bandwidth.* Bandwidth of a modulated signal may be made smaller or larger than the original signal. Signal to noise ratio in the receiver which is a function of the signal bandwidth can thus be improved by proper control of bandwidth at the modulating stage.

(*c*) *Shifting signal frequency to an assigned value.* The modulation process permits changing the signal frequency to a preassigned band. This frequency may be changed many times by successive modulations.

Other reasons why modulation is done include:

1. Avoidance of interference
2. Increase in the range of communication
3. Possibility of multiplexing signals
4. Improves quality of reception due to improves Signal-to- Noise ratio

The advantage of modulation which leads to the practicality of antenna design is briefly discussed below.

Whenever we transmit through space as a channel, there is need for installation of antennae for transmitting and reception of modulated signals. One question that normally comes to mind is "what happens if the signal is transmitted without modulation?" The minimum height of the antenna is given as $^1/_4$ of wave

Antenna height (or length) = ¼ λ

$$L = {}^{\lambda}/_{4} = {}^{V}/_{4f}$$

If we transmit with a base band signal of 3khz (i.e without modulation)

We have $L = {}^{3 * 10^3}/_{(4 * 3000)} = 25Km$

Obviously it is practically impossible to construct an antenna with length of 25000 meters.

If we now modulate with a signal of 3MHZ, then

$L = {}^{3 * 10^3}/_{(4 * 3,000,000)} = 25m$. It is practically possible to construct an antenna of length 25m.

Modulation may be defined as the process by which some parameter of a high frequency signal termed as carrier, is varied in accordance with the signal to be transmitted. Various madulation methods have been developed for transmission of signals as effectively as possible, with minimum possible distortion. The comparison of the effectiveness of these modulation methods may be based upon the signal power to noise power measured at the output of a receiver. Accordingly, a wide range of modulation techniques have been developed. These techniques may be broadly grouped into analogue techniques and pulse techniques. The analogue methods of modulation are simpler and cheaper than pulse modulation techniques. The former employ sinusoidal signals as carrier while the latter circuits use trains of pulses as the carrier signal.

Analogue modulation may be divided into *amplitude modulation and angle modulation.* Amplitude modulation (AM) may be categorised as AM with both side bands and carrier (AM/DSB), vestigial side band (VSB), double side-band suppressed carrier (DSB/SC),single side-band suppressed carrier (SSB/SC), and independent side-band suppressed carrier (ISB/SC).

AM/DSB is very popular for radio broadcast and radio telephony. For TV transmission with a large bandwidth, VSB is preferred because of reduced bandwidth of this modulation system. DSB/SC or SSB/SC provide a further reduction in power and bandwidth requirement. SSB/SC finds an extensive use in multiplexed coaxial system and can carry several messages simultaneously.

All AM systems are, however, prone to noise which directly affect the signal amplitude.

In angle modulation, the instantaneous angle of a sinusoidal carrier is varied as per the instantaneous amplitude of the modulating signal. The system leads to *phase modulation* (PM) and *frequency modulation* (FM). FM and PM waves require a much larger bandwidth than AM, but are capable of giving a sufficiently improved signal to noise ratio than the latter. It also leads to considerable saving in power.

Pulse modulation methods employ a pulse train as the carrier. The simplest type of pulse modulation is *pulse amplitude modulation* (PAM) which is similar to AM. Other pulse modulation techniques include *pulse duration modulation* (PDM), *pulse position modulation* (PPM) and *pulse code modulation* (PCM). While PAM may be compared with AM, PDM and PPM with angle modulation, The PCM has no analogue equivalent. The PCM have been developed as like *delta modulation* (DM) and *adaptive delta modulation* (ADM).

A third form of modulation consists of modulating a sinusodial signal with pulse signals and may be termed as *digital modulation*. Digital modulation may be divided into *amplitude shift keying* (ASK), *frequency shift keying* (FSK) and *phase shift keying* (PSK). They are especially useful for data transmission systems.

3.2. Amplitude Modulation

The process of amplitude modulation consists of varying the peak amplitude of a sinusoidal carrier wave in proportion to the instantaneous amplitude of the modulation signal. Though the modulating signal is usually an audio wave of speech or music which is complex in nature, the analysis of the AM system as well as others is restricted to a modulating signal that is sinusoidal and has single frequency.

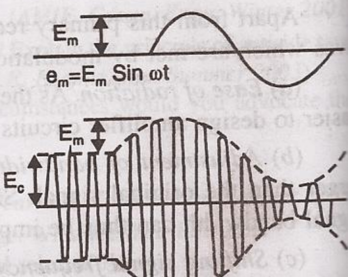


Fig. 3.1. Modulating signal and the AM wave.

Assume the modulating signal and the carrier be represented by $e_m = E_m \sin \omega_m t$ and $e_c = E_{cm} \sin \omega_c t$ where w_m and ω_c are the signal and the carrier angular velocities respectively. These components along with the resulting modulated waves are shown in Fig. 3.1.

The process of modulation increases the peak amplitude of the carrier so that for the modulated wave peak amplitude is given as

$E_{mod} = E_{cm} + E_m \sin \omega_m t$ and its instantaneous value is given as $e_{mod} = E_{mod} \sin \omega_c t$

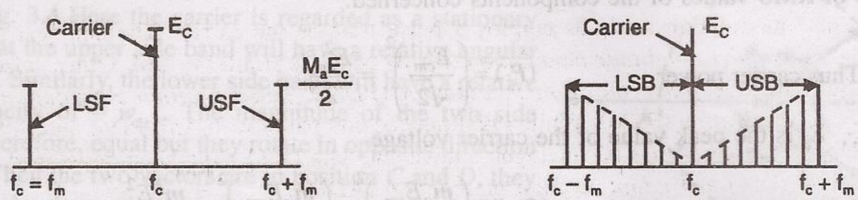
$$= (E_{cm} + E_m \sin \omega_c t) \sin \omega_c t$$

$$= E_{cm} \sin \omega_c t + E_m \sin \omega_m t \cdot \sin \omega_c t \quad \text{... (3.1)}$$

Expansion of Eq. (3.1) gives

$$e_{\text{mod}} = E_{cm} \sin \omega_c t + \frac{m_a}{2} E_{cm} \cos(\omega_c - \omega_m) t - \frac{m_a}{2} E_{cm} \cos(\omega_c + \omega_m) t \quad \text{... (3.2)}$$

where $m_a = \dfrac{E_m}{E_{cm}}$ is termed as degree of modulation. Equation 3.2 shows that the AM wave consists of three components $-E_{cm} \sin \omega_c t$ is the original carrier that is undisturbed, $\dfrac{m_a E_{cm}}{2} \cos(\omega_c - \omega_m)$ is the component that has peak amplitude $m_a E_c / 2$ and a frequency that is the difference between the carrier and the modulating signal and lastly the component $-m_a E_{cm} / 2 \cos(\omega_c + \omega_m) t$ is the component that has an amplitude equal to the second component but opposite in phase and a frequency that is the sum of the carrier and signal frequencies. Consequently, these two components are termed as Lower Side Band (LSB) and Upper Side Band (USB). Each of the side bands is equally spaced from the carrier frequency and has a magnitude that 0.5 m_a times the carrier amplitude E_c. Frequency spectrum of the AM wave so obtained is shown in Fig. 3.2 (a) for a single modulating frequency and for a band of frequencies in Fig. 3.2 (*b*).



(*a*) Upper and lower sidebands for single frequency AM wave.
(*b*) USB and LSB for a band of modulation signal frequencies.

Fig. 3.2.

One cycle of the modulating sine wave is redrawn in here (Fig 3.3). The top envelope of the AM wave gives the relation $E_c + E_m \sin \omega_m t$. Similarly the bottom envelope gives the relation $-(E_c + E_m \sin \omega_m t)$. The modulated wave extends between these two limiting envelopes and has a repetition rate equal to the unmodulated carrier frequency. The depth of modulation is given by

$ma = \frac{E_m}{E_c}$. The depth of modulation is also calculated in terms of E_{max} and E_{min}. As seen from the Fig. 3.3.

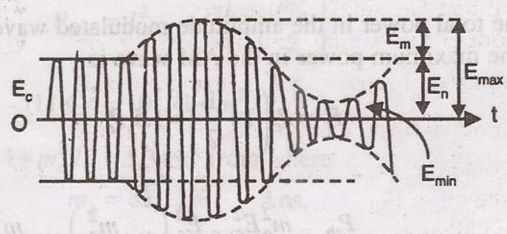


Fig. 3.3. Amplitude-modulated wave

$$E_{max} = E_{min} + E_m + E_m$$

or
$$2\,E_m = E_{max} - E_{min}$$

$$E_m = \frac{E_{max} - E_{min}}{2} \quad \text{.... (3.3)}$$

also
$$E_C = E_{max} - E_m$$

Putting value of E_m from Eq. 3.3

$$E_C = E_{max} - \frac{E_{max} - E_{min}}{2}$$

$$E_C = \frac{E_{max} + E_{min}}{2} \quad \text{... (3.4)}$$

Dividing Eq. 3.3 by Eq. 3.4, we have

$$m_a = \frac{E_m}{E_C} = \frac{(E_{max} - E_{min})/2}{(E_{max} + E_{min})/2}$$

$$m_a = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} \quad \text{... (3.5)}$$

Eq. 3.5 gives the standard method of evaluating the depth of modulation or modulation index when AM wave is displayed on an oscilloscope.

3.2.1. Power in an AM Wave. Consider the wave as represented by Eq. 3.2 being fed to a load resistor of 1 ohm. The average power in each of the component of the wave is given by the square of RMS values of the components concerned.

Thus carrier power $$(P_c) = \left(\frac{E_{cm}}{\sqrt{2}}\right)^2 = \frac{E_{cm}^2}{2}$$

$\therefore$ E_c is the peak value of the carrier voltage.

Power in side bands $$(P_{SB}) = \left(\frac{m_a E_{cm}}{2\sqrt{2}}\right)^2 + \left(\frac{m_a E_{cm}}{2\sqrt{2}}\right)^2 = \frac{m_a^2 E_{cm}^2}{4}$$

Total power, $$= \frac{E_{cm}^2}{2}\left(1+\frac{m_a^2}{2}\right)$$

If the carrier voltage has a RMS value E_c; then total power

$$(P_t) = E_c^2\left(1+\frac{m_a^2}{2}\right) = P_c\left(1+\frac{m_a^2}{2}\right)$$

$$\frac{P_t}{P_c} = 1+\frac{m_a^2}{2} \quad \text{.... (3.6)}$$

Eq. 3.6 relates the total power in the amplitude modulated wave to the unmodulated carrier power. When $m_a = 1$ the maximum power in the AM wave is

$$P_t = P_c\left(1+\frac{1}{2}\right) = 1.5\,P_c$$

Similarly

$$\frac{P_{sb}}{P_t} = \frac{m_a^2 E_c^2}{4} \div \frac{E_c^2}{2}\left(1+\frac{m_a^2}{2}\right) = \frac{m_a^2}{2+m_a^2} \quad \text{.... (3.7)}$$

When percentage modulation is 100%, $m_a = 1$ and equation 3.7, becomes $\frac{P_{sb}}{P_t} = \frac{1}{3}$.

As can be seen for $m_a = 1$, power in side bands is only 1/3 of the total power in the AM wave. The remaining two-third power is in the carrier. Since the carrier does not contain any intelligence, it follows that an AM wave has only 1/3 of its power as useful for $m_a = 1$. In practice, however, the value of m_a lies between 0.3 to 0.5 and useful power varies from 4.3% to 11.1% of the total power.

Current relation

The depth of modulation can also be calculated from current relations. Let I_c (rms) be the unmodulated current and I_t (rms) is the total or modulated current of an AM transmitter. Let R is the resistance in which these currents flows, then

$$\frac{P_t}{P_c} = \frac{I_t^2 . R}{I_c^2 . R} = \left(\frac{I_t}{I_c}\right)^2$$

$$= 1+\frac{m_a^2}{2}$$

or

$$\frac{I_t}{I_c} = \sqrt{1 + \frac{m_a^2}{2}} \quad \text{.... (3.8)}$$

3.2.2. Vector Representations. Since AM wave is a vector quantity, it may be represented by rotating phasor diagrams as shown in Fig. 3.4 Here the carrier is regarded as a stationary phasor so that the upper side band will have a relative angular velocity w_m. Similarly, the lower side band will have a relative angular velocity of $-w_m$. The magnitude of the two side bands are, therefore, equal but they rotate in opposite direction as shown. When the two vectors are in position C and D, they neutralize each other and the resultant voltage equals E_c. When both the phasors are in position A, the modulated wave has a minimum magnitude of $(E_c - E_m)$. Similarly when both the phasors are in position B, the wave has a maximum magnitude of $(E_c + E_m)$.

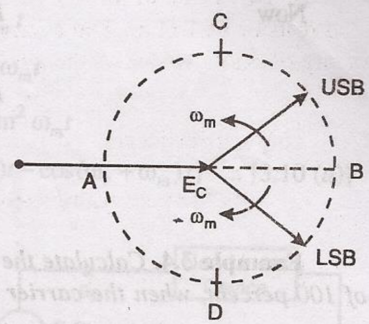


Fig. 3.4. Phasor representation of an amplitude modulated wave.

Example 3.1. *In an aerial, the aerial current (RMS) before modulation is 10 amps. After modulation, it rises to 11.6 amps. Determine percentage modulation. If the carrier power is 10 kW, what is the power after modulation ?*

Solution. Carrier current $I_c = 10$ amp.

Current after modulation = $I_{mod} = 11.6$ amp.

If the load is assumed to be 1 Ohm, then

$$P_{mod} = I_{mod}^2 = I_c^2 \left(1 + \frac{m_a^2}{2}\right)$$

$$(11.6)^2 = 10^2 . (1 + m_a^2 / 2)$$

or $$1 + m_a^2/2 = 1.3456 \text{ from where}$$

$$m_a = 83.13\% \quad \textbf{Ans.}$$

Now $$P_{mod} = P_c (1 + m_a^2/2)$$

$$P_{mod} = 10 \text{ kW } (1.3456)$$

$$= 3.456 \text{ kW} \quad \textbf{Ans.} \quad m_a = 83.13\%$$

$$P_{mod} = 13.456 \text{ kW}$$

Example 3.2. *A transmitter radiates 9 kW without modulation and 10.125 kW after modulation. Determine depth of modulation.*

Solution. $P_c = 9$ kW

$P_{mod} = 10.125$ kW

Now $$P_{mod} = P_c (1 + m_a^2/2)$$

$$10.125 = 9 \text{ kW } (1 + m_a^2/2)$$

or $$1 + m_a^2/2 = 1.125$$

$$m_a = 0.5 \quad \textbf{Ans.}$$

Example 3.3. *A transmitter radiates 1200 watts of power under carrier conditions. If this carrier is modulated simultaneously by two tones of 20% and 40% respectively, determine the total power radiated.*

Solution. Let depth of modulation be m_1 and m_2

$$m_1 = 20\% = 0.2$$

and $$m_2 = 40\% = 0.4$$

Now $$P_{mod} = P_c(1 + m_1^2/2 + m_2^2/2 + ...)$$

$$P_{mod} = 1200\left(1 + \frac{0.2^2}{2} + \frac{0.4^2}{2}\right)$$

$$= 1200\ (1 + 0.02 + 0.08)$$
$$= 1200 \times 1.1$$
$$= 1320 \text{ watts. } \textbf{Ans.}$$

Example 3.4. *Calculate the percentage power saving in an AM modulated wave to a depth of 100 percent, when the carrier and one of the sidebands are suppressed.*

Solution. Total power $$P_t = P_c\left(1 + \frac{m_a^2}{2}\right), \quad m_a = 1$$

then $$P_t = 1.5\ P_c$$

$$P_{sb} = P_c\frac{m_a^2}{4} = 0.25\ P_c$$

$$\text{Saving } = \frac{P_t - P_{sb}}{P_t} = \frac{1.5 - 0.25}{1.5}$$

$$= 0.833$$
$$= 83.3\% \quad \textbf{Ans.}$$

A saving of two-thirds of power when depth of modulation is 100 percent.

3.2.3. Amplitude Modulation Circuits. There is a variety of modulator circuits employing tubes or solid state devices to produce amplitude modulated waves. All these circuits may, however, be grouped as (*a*) Square Law or Non-linear modulator circuits and (*b*) Linear modulator circuits. Another type of grouping is also sometimes done. This is based upon the power level at which modulation is carried out and may be termed as (*a*) Low level modulation and (*b*) High level modulation. In low level modulation, the modulation is carried out at a low power level. In high level modulation, the modulation is done at high power levels.

It is worthwhile to note that in general Square law modulators are low level modulators while Linear modulators are high level modulators.

Non-linear Modulation

In general, any device operated in non-linear region of its output characteristic is capable of producing amplitude modulated waves when the carrier and the modulating signals are fed at the input. Thus, a transistor, a triode tube, a diode etc., may be used as a square law modulator. In such a modulator circuit, the output current flowing through the load is given by the power series.

$$i = a_0 + a_1 e_1 + a_2\ e_1^2 + ... \qquad .... (3.9)$$

where a_0, a_1, a_2 etc., are constants and e_1 is the input voltage to the device. Considering the modulator circuit of Fig. 3.5 (*a*).

$$e_1 = E_c \sin \omega_c t + E_m \sin \omega_m t$$

$$\therefore \quad i = a_0 + a_1\ (E_c \sin \omega_c\ t + E_m \sin \omega_m\ t)$$
$$+ a_2(E_c \sin \omega_c\ t + E_m \sin \omega_m\ t)^2 + ... \qquad ... (3.10)$$

$$= a_0 + a_1 E_c \sin\omega_c\, t + a_1 E_m \sin\omega_m\, t$$
$$+ a_2\, E_c^2 \sin^2 \omega_c\, t + a_2\, E_m^2 \sin^2 \omega_m t$$
$$+ 2a_2\, E_c E_m \sin\omega_c\, t \sin\omega_m\, t$$
$$= a_0 + a_1\, E_c \sin\omega_c t + a_1 E_m \sin\omega_m t$$
$$+ a_2 E_c^2 \sin^2 \omega_c t + a_2 E_m^2 \sin^2 \omega_m t$$
$$+ a_2\, E_c E_m \{\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\} \quad \ldots [3.10\ (a)]$$

The last term of Eq. 3.10 (*a*) gives the upper and lower side-bands while the second term gives the carrier. If the load is a resonant circuit, side-bands and carrier may be selected giving the AM outpout.

When all unwanted frequencies are rejected, the modulated component present at the output is represented by

$$i = a_1 E_c \sin\omega_c t + a_2 E_c \cdot E_m \{\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\}$$

As a_1 is considerably larger than , the depth of modulation that is available without distortion is low. Also note that the circuit efficiency is quite low because a sufficient number of components are filtered out from the plate current. Thus this type of circuit is essentially a low level modulation circuit.

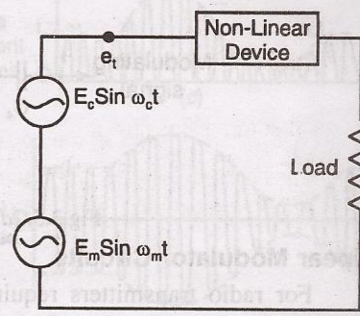
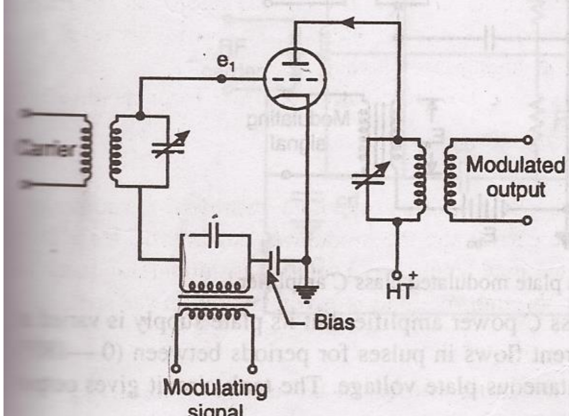
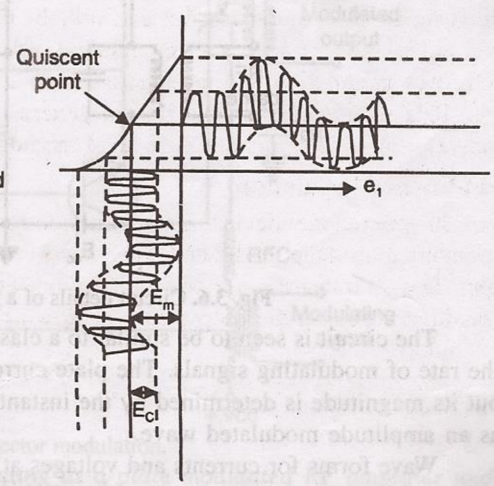


Fig. 3.5. (*a*). **Basic non-linear modulator.**

A common circuit using the square law modulation technique is the *Van Der Bijl Modulator circuit* shown in Fig. 3.5 (*b*) while its wave forms are shown in Fig. 3.5 (*c*). A Van Der Bijl modulator using transistor is shown in Fig. 3.5 (*d*). The circuit is connected in common emitter configuration. The modulating signal is applied to the emitter and *RF* carrier at the base of the transistor. The transistor used is the switching transistor which operates in the non-linear transfer characterstics. It is biased in such a way that it operates in the class *A*-mode and efficiency is low. This circuit is used in the low level modulation.



(*b*) Van Der Bijl modulator.



(*c*) Wave forms for Van Der Bijl modulator

Fig. 3.5.

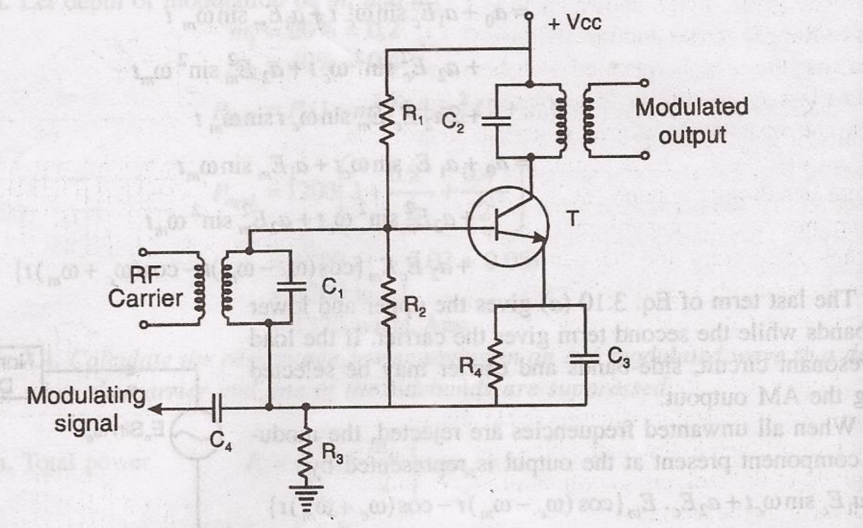


Fig.3.5. (*d*) Van Der Bijl modulator using transistor

Linear Modulator Circuits

For radio transmitters requiring good linearity and high power output, linear modulator circuit in the form of a class *C* plate modulated triode amplifier is very commonly employed. The modulating signals should also have a comparable power and for this purpose class *B* audio power amplifiers are used. This is a widely used high level modulation circuit that is capable of 100% modulation with low distortion of the order of 2%. The circuit has a high efficiency and requires simple adjustments. However, it requires high voltages from the carrier and the audio circuits. Figure 3.6 gives the circuit arrangements of a plate modulated class *C* amplifier circuit.

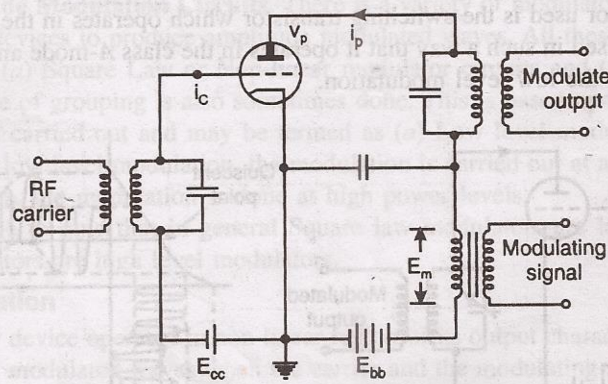


Fig. 3.6. Circuit details of a plate modulated class *C* amplifier.

The circuit is seen to be similar to a class *C* power amplifier but its plate supply is varied at the rate of modulating signals. The plate current flows in pulses for periods between (0 —180°) but its magnitude is determined by the instantaneous plate voltage. The tank circuit gives output as an amplitude modulated wave.

Wave forms for currents and voltages at various points are shown in Fig. 3.7. The instantaneous plate potential e_p is shown in Fig. 3.7 (*a*) and is seen to be the algebraic sum of the supply voltage E_{bb} and the modulating signal E_m.

The grid voltage shown in Fig. 3.7 (*b*) consists of the RF carrier and the DC bias E_{cc}. E_{cc} is so fixed that the tube is operated well beyond cut-off under class *C* conditions. The plate current flows when the instantaneous grid potential crosses the grid cut-off line E_{co}. As a result, the angle of plate current lies in the range of 150°—180°. The grid is driven positive to ensure that the plate current touches saturation line. Under these conditions, when modulating signal is applied, the saturation moves up and down every time the anode current reaches this point thereby giving constant efficiency.

Figure 3.7 (*c*) gives the pulses of plate current during the carrier and modulation cycle and are seen to be proportional to the plate voltage. These pulses are given to the tank circuit which produces modulated wave at the secondary.

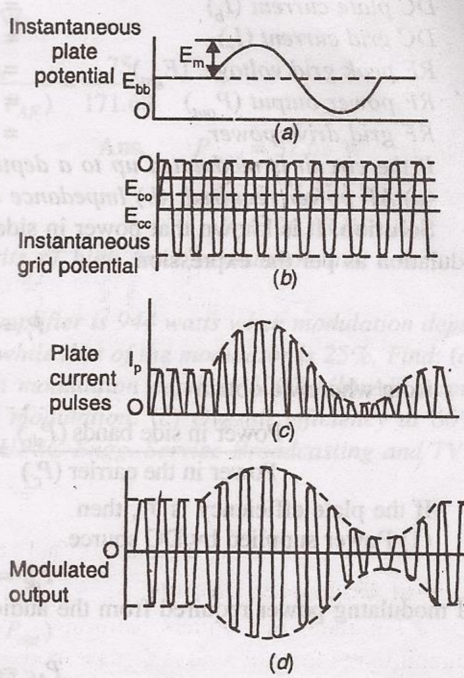


Fig. 3.7. Modulated output wave forms.

Collector Modulation

A transistor collector modulation circuit is used for low power-level modulation AM transmitters. A collector modulation circuit is shown in Fig. 3.8. The modulating signal is applied to the collector and RF carrier at the bases of the transistors. The modulated output is obtained at the collector. The collector modulator has higher collector efficiency and higher power output per transistor.

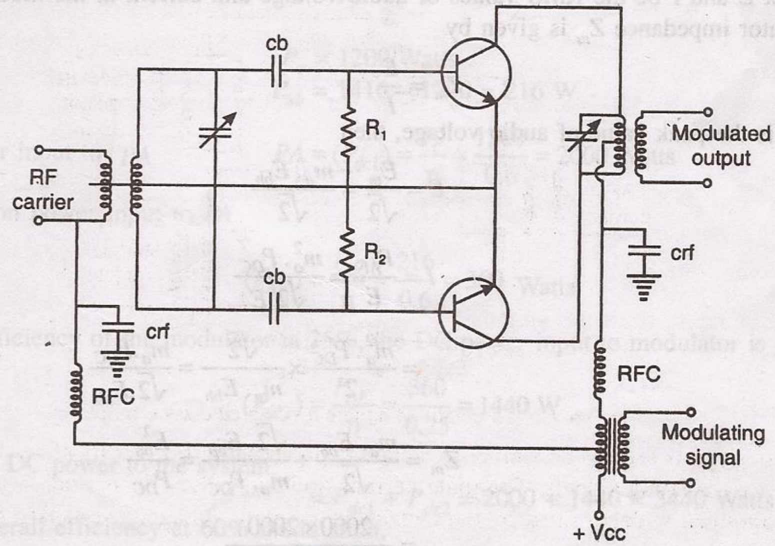


Fig. 3.8. Collector modulation.

Example 3.5. A *triode has the following rating as a plate modulated RF amplifier under carrier conditions for 100% modulation.*

DC plate supply (E_{bb}) = *2 kV*

DC grid bias (E_{cc}) = *– 500 V*

DC plate current (I_b) $= 67\ mA$
DC grid current (I_c) $= 30\ mA$
RF peak grid voltage, (E_{gm}) $= 750\ V$
RF power output (P_{out}) $= 75\ W$
RF grid drive power $= 23\ W$

If the circuit is modulated up to a depth of 75%, calculate:

(a) AF power required; (b) Impedance offered to audio source; and (c) plate dissipation.

Solution. It is known that power in side bands is related to the carrier power and depth of modulation as per the expression.

$$P_t = P_c\left(1+\frac{m_a^2}{2}\right)$$

from where we obtain

$$\frac{\text{Power in side bands } (P_{sb})}{\text{Power in the carrier } (P_c)} = \frac{m_a^2 . P_c / 2}{P_c} = \frac{m_a^2}{2}$$

If the plate efficiency is η, then

(1) Power supplied by DC source

$$P_{dc} = P_c / \eta$$

and modulatng power required from the audio source is

$$P_{AF} = P_{sb} / \eta = \frac{m_a^2 . P_c}{2\eta} = \frac{m_a^2 . P_{DC}}{2} = \frac{m_a^2\ E_{bb} . I_b}{2}$$

$$= \frac{0.75 \times 0.75 \times 2000 \times 67 \times 10^{-3}}{2}$$

$$= 37.68 \text{ W}$$

(*b*) Let E and I be the RMS values of audio voltage and current at the modulator output, then modulator impedance Z_m is given by

$$Z_m = \frac{E}{I}$$

If E_m is the peak value of audio voltage, then

$$E = \frac{E_m}{\sqrt{2}} = \frac{m_a . E_{bb}}{\sqrt{2}}$$

But

$$I = \frac{P_{AF}}{E} = \frac{m_a^2 . P_{DC}}{\sqrt{2}\, E}$$

$$= \frac{m_a^2 . P_{DC}}{2} \times \frac{\sqrt{2}}{m_a . E_{bb}} = \frac{m_a . P_{DC}}{\sqrt{2} . E_{bb}}$$

$$\therefore \quad Z_m = \frac{m_a . E_{bb}}{\sqrt{2}} \div \frac{\sqrt{2} . E_{bb}}{m_a . P_{DC}} = \frac{E_{bb}^2}{P_{DC}}$$

$$= \frac{2000 \times 2000}{2000 \times 67 \times 10^{-3}}$$

$$= 29.84 \text{ K}\Omega.$$

(c) Plate dissipation $= P_{DC} + P_{AF} - P_{out}$

$= 2000 \times 67 \times 10^{-3} + 37.68 - 75$

$$= 96.68 \text{ W}$$

and $$\eta = \frac{P_{out}}{(P_{DC} + P_{AF})} = \frac{75}{171.68}$$

$$= 43.7\% \quad \textbf{Ans.} \quad P_{mod} = 37.68 \text{ W}$$

$$Z_m = 29.84 \text{ K}\Omega$$

$$P_d = 96.68 \text{ W}$$

Example 3.6. *(a). What are the relative merits of high level modulation and low level modulation in AM transmission ?*

(b) The anode dissipation of a class C power amplifier is 944 watts when modulation depth is 60%, the efficiency of the power amplifier is 60%, while that of the modulator is 25%. Find: (a) Carrier power and modulator tube dissipation when modulation depth is 100%. (b) AF output and rating of the modulation value to affect 100% modulation. (c) Overall efficiency at 60% modulation depth. *(UPSC Engg. Service Broadcasting and TV)*

Solution. For part (*a*) see text.

(*b*) Power dissipation at 60% modulation,

$$(P_d) = 944 \text{ W}$$

Now $$\eta = 60\%$$

$$\therefore \quad P_{out} = \eta\,(P_d + P_{out})$$

or $$P_{out} - 0.6\,P_{out} = 0.6 \times 944$$

$$P_{out} = 1416 \text{ W}$$

At $m_a = 0.6$, $$P_c = \frac{P_t}{1 + \frac{m_a^2}{2}} = \frac{1416}{1 + \frac{0.6 \times 0.6}{2}}$$

or $$P_c = 1200 \text{ Watts}$$

$$\therefore \quad P_{sb} = 1416 - 1200 = 216 \text{ W}$$

DC power input to PA $$PA = (P_{dc1}) = \frac{P_c}{\eta} = \frac{1200}{0.6} = 2000 \text{ Watts}$$

Modulation power input to PA

$$(P_{AF}) = \frac{P_{sb}}{\eta} = \frac{216}{0.6} = 360 \text{ Watts}$$

As the efficiency of the modulator is 25%, the DC power input to modulator is given by

$$(P_{dc2}) = \frac{P_{AF}}{\eta} = \frac{360}{0.25} = 1440 \text{ W}$$

$\therefore$ Total DC power to the system,

$$= P_{dc1} + P_{dc2} = 2000 + 1440 = 3440 \text{ Watts}$$

Hence overall efficiency at 60% modulation,

$$\eta = \frac{1416}{3440} \times 100 = 41.1\%$$

For 100% modulation, the carrier does not change but more modulated signal power is to be given.

$$\therefore \quad P_c = 1200 \text{ W and } P_t = P_c\left(1+\frac{m_a^2}{2}\right) = 1800 \text{ W}$$

and $$P_{sb} = \frac{P_c \times m_a^2}{2} = \frac{1200\times 1}{2} = 600 \text{ W}$$

Modulating power input to *PA*

$$P_{AF} = \frac{P_{sb}}{\eta} = \frac{600}{0.6} = 1000 \text{ W}$$

Since the efficiency of the modulator is 25%

$\therefore$ DC power input to modulator,

$$P_{dc2} = \frac{1000}{0.25} = 4000 \text{ W}$$

Power dissipation at the modulator,

$$= 4000 - 1000 = 3000 \text{ W}$$

Overall efficiency of the system,

$$= \frac{P_{out}}{P_{dc1}+P_{dc2}}\times 100$$

$$= \frac{1800\times 100}{2000+4000} = 30\%$$

Ans. $P_c = 1200$ W

$P_d = 3000$ W

$\eta = 41.1\%$

Example 3.7. *A class C plate modulated amplifier under 100% modulation with a sinusoidal signal requires a total plate input power of 1400 Watts and its plate dissipation rises to 400 Watts. Calculate (a) the carrier power output and plate dissipation when there is no modulation (b) Modulator power output and plate dissipation at 100% modulation if the plate efficiency of the modulator is 0.6.* *(UPSC Engg. Services Broadcasting and TV)*

Solution. Under 100% modulation, DC power input to power amplifier,

$$P_{dc} = 1400 \text{ W}$$

Plate dissipation = 400 W

Out of this power, DC plate dissipation,

$$= \frac{400\times 2}{3} = 266.67 \text{ Watts}$$

and plate dissipation from the modulating signal,

$$= \frac{400\times 1}{3} = 133.33 \text{ Watts}$$

Carrier power $(P_c) = 1400 - 266.67 = 1133.33$ Watts

$\therefore$ Side band power, $$(P_{sb}) = \frac{1133.33}{2} = 566.67 \text{ Watts}$$

The modulator must, therefore, provide a power output,

$$P_{AF} = 566.67 + 133.33 = 700 \text{ Watts}$$

As modulator efficiency is 60%, DC power input to the modulator is given by

$$P_{dc2} = \frac{700}{0.6} = 1166.67 \text{ W}$$

Plate dissipation in the modulator,

$$= 1166.67 - 700 = 466.67 \text{ Watts.} \quad \textbf{Ans.} \quad P_c = 1133.33 \text{ W}$$

$$P_d = 266.67 \text{ W}$$

$$P_{AF} = 700 \text{ W}$$

$$P_{d(AF)} = 466.67 \text{ W}$$

Example 3.8. *Discuss the merits and demerits of high level modulation with respect to low level modulation. A class C amplifier is amplitude modulated with a modulator capable of giving 500 Watts of AF power. The amplifier operates with an efficiency of 75%. What is the maximum carrier power from the amplifier for 100% modulation? Find the total RF power during 100% modulation.*

Solution. Modulator output power, $= P_{AF} = 500$ Watts

Modulating power lost in the amplifier, $= 500 \times 0.25 = 125$ W

and side band power $= P_{sb} = 500 \times 0.75 = 375$ W

As $m = 100\%,\ P_c = 2P_{sb}$

or $P_c = 750$ Watts

Total RF power $(P_t) = P_a + P_{sb} = 750 + 375$

$= 1125$ Watts. **Ans.** $P_c = 750$ Watts

$P = 1125$ Watts

For merits/demerits see text.

Example 3.9. *Draw the final stage of a typical AM transmitter that uses a push-pull modulation to modulate a class C plate modulated amplifier.*

A push-pull modulator stage comprises of two valves each rated for 750 Watts and operating at an efficiency of 55% is used to plate modulate a class C amplifier operating at an efficiency of 60%. The power amplifier is rated for 3000 Watts of RF output and is operating at its fully loaded conditions. Find the carrier power and the maximum depth of modulation.

Solution. For first part see text.

Rated power output from push-pull modulator,

$$= P_{AF} = 2 \times 750 = 1500 \text{ Watts}$$

Modulation power lost in, PA

$$= P_{AF} - P_{sb} = P_{AF} - \eta \,.\, P_{AF}$$

$$= 1500 - 0.6 \times 1500 = 600 \text{ Watts}$$

$$P_{sb} = 900 \text{ Watts}$$

Carrier power, $(P_c) = 3000 - 900 = 2100$ Watts

Now $$P_{sb} = \frac{m_a^2 P_c}{2}$$

$$m_a = \sqrt{\frac{2P_{sb}}{P_c}} = \sqrt{\frac{2 \times 900}{2100}} \quad \textbf{Ans.} \quad P_c = 2100 \text{ Watts}$$

$$m = 92.5\%$$

Example 3.10. *The output voltage of a transmitter is given by 500 (1 + 0.4 sin 3140 t) sin 6.28 t. This voltage is fed to a load of 600 Ω resistance. Determine (a) carrier frequency (b) modulating frequency (c) carrier power (d) mean power output (e) peak power output.*

Solution. $$e = 500(1 + 0.4 \sin 3140\, t) \sin 6.28 \times 10^7\, t$$

(*a*) $$w_c = 6.28 \times 10^7$$

or $$f_c = \frac{6.28 \times 10^7}{2\pi} = 10 \text{ MHz}$$

(b) $w_m = 3140$

or $f_m = 3140/6.28 = 500 \text{ Hz}$

(c) Peak carrier voltage, = 500 V

Carrier power P_c across 600 W load

$$= \left(\frac{500}{\sqrt{2}}\right)^2 \Big/ 600 = \frac{2500}{12} = 208.33 \text{ W}$$

(d) Mean power output,

$$(P_t) = P_c\left(1 + \frac{m_a^2}{2}\right)$$

$$= \frac{2500}{12}\left(1 + \frac{(0.4)^2}{2}\right)$$

$$= \frac{2500}{12} \times 1.08 = 225 \text{ Watts}.$$

(e) Peak power output results when the positive half-cycle of the modulating signal occurs. The peak output voltage is given by the sum of E_c and E_m.

Peak output voltage = $E_c + m_a E_c$

$= 500 + 0.4 \times 500 = 700$ V

Peak power $P_{tm} = \frac{700}{\sqrt{2}} \times \frac{700}{\sqrt{2}} \times \frac{1}{600} = 408.3 \text{ Watts}$

Example 3.11. *(a) Show a typical arrangement for plate modulating an RF amplifier. Explain the function of all the components used.*

(b) A transmitter is adjusted to deliver 50 kW of carrier power to an antenna whose base impedance is 36 + j 40 ohms.

(i) What will be the antenna base current when the transmitter is held at a sustained tone modulation of 100% ?

(ii) What will be the peak voltage appearing across the base insulator at the crest of modulation.?

(iii) If the modulation is reduced to 50% what will be peak voltage across the base insulator?

Solution. For part (a) see text.

(b) Carrier power $'P_c' = 50 \text{ kW}$

Under 100% modulation, Power delivered to the antenna

$$= P_{mod} = P_c\left(1 + \frac{m_a^2}{2}\right) = 50 \text{ kW}\left(1 + \frac{1}{2}\right) = 75 \text{ kW}$$

(i) Since this power is dissipated across the resistance part of the antenna impedance, then the antenna current

$$I_{RMS} = \sqrt{\frac{\text{Power}}{R}}$$

$$= \sqrt{\frac{75000}{36}} = 45.64 \text{ amps}$$

Ans. $\left.\begin{array}{l}\text{Ant. Current}\\ \text{for } M = 100\%\end{array}\right\} = 45.64 \text{ amps}$

(*ii*) RMS current under carrier condition,

$$= I_c = \sqrt{\frac{P_c}{R}} = \sqrt{\frac{50000}{36}} = 37.27 \text{ amps}$$

Peak carrier current $\quad = I_{cm} = 37.27 \times \sqrt{2} = 51.5$ amps

With 100% modulation, the current I_{cm} becomes doubled *i.e.*,

$$I_{mod} = 51.5 \times 2 = 103 \text{ amp.}$$

Peak voltage across the base insulator during crest of modulation,

$$= I_{mod}\,(Z_{ant}) = 103(36 + j\,40)$$

$$= 5541.4 \;\angle 48° \text{ volts}$$

Ans. Peak base voltage for 100% mod = 5541.4 $\angle$ 48°.

(*iii*) With 50% modulation, the peak current at the antenna input,

$$I_{mod} = 51.5 + \frac{51.5}{2} = 77.25 \text{ amps}$$

Peak voltage across the base modulator during crest of modulation,

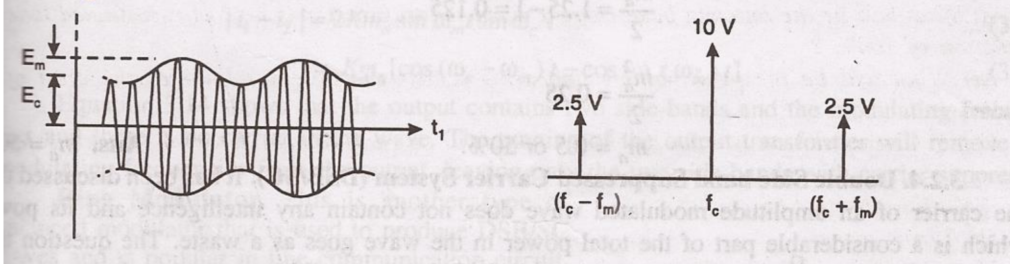
$$= I_{mod}\,(Z_{ant}) = 77.25 \times (36 + j\,40)$$

$$= 4156.05 \angle 48°$$

Ans. Peak base voltage for 50% mod = $= 4156.05 \angle 48°$

Example 3.12. *A carrier wave of 10 MHz is amplitude modulated to 50% level with a tone 5000 Hz. Sketch the wave form and amplitude distribution in time and frequency domain. Assuming carrier amplitude of 10 volts, write down equation of the above wave and show existence of two side bands. Determine their frequency and amplitude.*

Solution. Wave form and amplitude distribution of the modulation wave are shown in Fig. (*a*) and (*b*).



(*a*) Wave form of the AM wave with 50% modulation in time domain

(*b*) Amplitude distribution in frequency domain

Fig. 3.9

Equation of an AM wave is

$$e = E_c(1 + m_a \sin \omega_m\, t) \sin \omega_c\, t$$

In this case, $\quad E_c = 10\,V,\ m_a = 0.5,\ \omega_m = 10000\pi$ and $\omega_c = 2\pi \times 10^7$

$$e = 10\,[1 + 0.5 \sin 10000\,\pi t] \sin 2\pi \times 10^7\, t$$

$$= 10 \sin 2\pi \times 10^7\, t + 2.5 \cos 2\pi\,(10^7 - 50000)\, t$$

$$-2.5 \cos 2\pi\,(10^7 + 500)\, t$$

The two side bands are,

U.S.B. frequency $= 10^7 + 5000 = 10005$ KHz

L.S.B. frequency $= 10^7 - 500 = 9995$ KHz

U.S.B. amplitude $= 2.5$ V

L.S.B. amplitude $= 2.5$ V

Carrier amplitude $= 10$ V

Ans. U.S.B. frequency = 10005 KHz
U.S.B. amplitude = 2.5 V
L.S.B. frequency = 9995 KHz
L.S.B. amplitude = 2.5 V
Carrier frequency = 10 V.

Example 3.13. (*a*) *Explain the term amplitude modulation of a carrier wave. Illustrate with the help of simple sketches 50% and 100% modulation.*

(*b*) *A transmitter radiates 9 kW of power with carrier unmodulated and 10.125 kW when modulated. Calculate the depth of modulation.* (*Nagpur University, B.E. VI Sem. Winter 2004*)

Solution. For part (*a*) see text.

(*b*) $$P_t = P_c\left(1+\frac{m_a^2}{2}\right)$$

Now $P_c = 9$ kW

and $P_t = 10.125$ kW

$$10.125 = 9\left(1+\frac{m_a^2}{2}\right)$$

$$1+\frac{m_a^2}{2} = \frac{10.125}{9} = 1.125$$

$$\frac{m_a^2}{2} = 1.25 - 1 = 0.125$$

$$\frac{m_a^2}{2} = 0.25$$

$$m_a = 0.5 \text{ or } 50\%.$$

Ans. $m_a = 50\%$

3.2.4. Double Side band Suppressed Carrier System (DSB/SC). It has been discussed that the carrier of an amplitude modulated wave does not contain any intelligence and its power which is a considerable part of the total power in the wave goes as a waste. The question that naturally arises is why transmit the carrier at all. In fact, it is possible not to transmit the carrier at all and transmit only the side-bands. This will lead to a considerable power saving *i.e.* two-third of the total power if modulation depth is 100%.

However, the suppression of the carrier from a transmitted wave leads to a considerably complicated design at the receiving end. As a result, the receiver becomes costlier. If the waves are used for broadcast work involving a large number of receivers, the additional expenditures on these receivers outweighs the advantages of power saving resulting from the suppressed carrier at the transmitter. Therefore, it has become the general practice to employ DSB—AM for broadcast service and utilise suppressed carrier systems for radio telephone links only.

Alternately, instead of suppressing the carrier completely it is possible to suppress the carrier partially and transmit it at reduced power level. But another question that immediately comes to mind is—what is the use of sending both the side bands when even a single side band would suffice the needs of the communication links. In fact, it has become the standard practice in communication circuits to transmit only a single side band with a very low level carrier. The carrier is gnerally termed as the *pilot carrier* and the system is called SSB/SC.

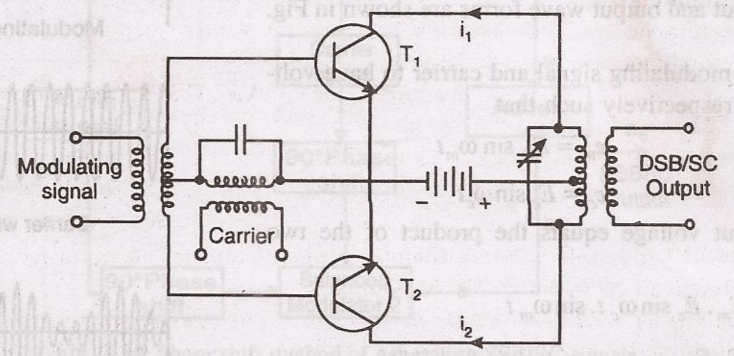


Fig. 3.10. A transistorised push-pull balanced modulator.

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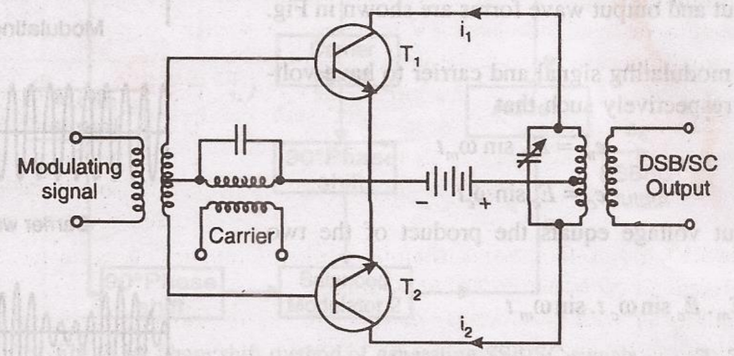


Fig. 3.10. A transistorised push-pull balanced modulator.

Ring Modulator. This is another type of balanced modulator that is used to produce DSB/SC waves and is popular in line communication circuit. Figure 3.11 shows the circuit arrangement of a ring modulator. The circuit employs diodes as non-linear devices and the carrier signal is connected between the centre taps of the input and output transformers.

To understand the functioning of this circuit, consider the case when there is no carrier and modulating signal is present. Diodes D_1, D_2 or D_3, D_4 will conduct depending on the signal polarity and will provide an effective short circuit thereby prohibiting the signal from reaching the output. Similarly, when carrier alone is present, the flow of current in two-halves of the output transformer is equal and opposite and no output can develop across the load.

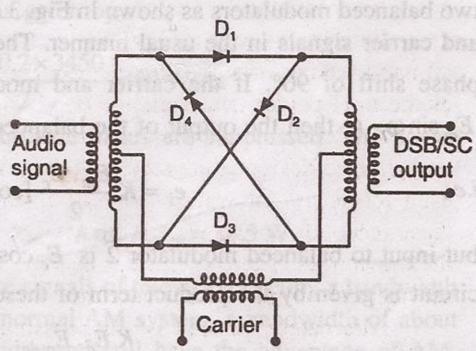


Fig. 3.11. A Ring modulator to produce DSB/SC.

When both the carrier and the modulating signals are present, the resultant potential in one-half of the output transformer becomes larger than the other and output is obtained. Input and output wave forms are shown in Fig. 3.12.

Consider modulating signal and carrier to have voltages e_m and e_c respectively such that

$$e_m = E_m \sin \omega_m t$$

and
$$e_c = E_c \sin \omega_c t$$

The output voltage equals the product of the two signals.

$$\therefore\ e_o = E_m . E_c \sin \omega_c t . \sin \omega_m t$$

$$= \frac{E_m E_c}{2} \{\cos (\omega_c - \omega_m) t - \cos (\omega_c + \omega_m) t\} \quad \text{... (3.15)}$$

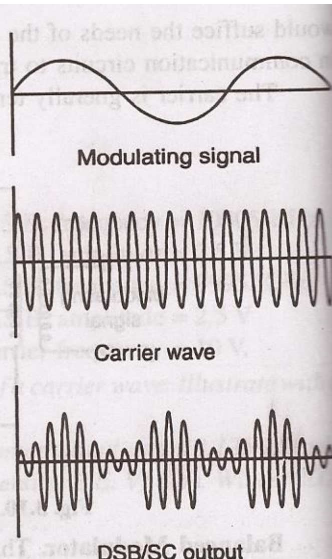


Fig. 3.12. Input and output wave forms of a balanced modulator

The output given by Eq. 3.15 is seen to be free from the carrier and contains upper and lower side bands only. The ring modulator circuit is also known as *double balanced modulator.*

3.2.5. Single Side band Suppressed Carrier (SSB/SC). As has been pointed out a single side band is sufficient to convey information to the distant receiver. This reduces the power and bandwidth requirements considerably. Moreover, the suppressed side bands may be used as a second independent communication channel.

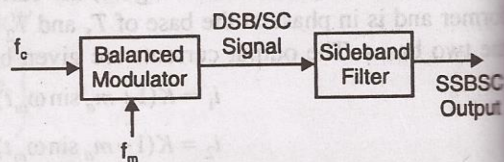


Fig. 3.13. Filter method of producing SSB/SC signals.

A simple arrangement to produce SSB/SC signals is the use of a filter for filtering out one of the side bands leaving at the output the other side band alone. The arrangement is shown in Fig. 3.13.

However, in practice, it is difficult to design a filter with sharp cut-off on either side. If the bandwidth is reduced in an effort to eliminate the unwanted side band, such a filter will introduce attenuation in the wanted side band also. Increasing the bandwidth may result in passing some of the unwanted side bands to the output.

Another way of producing a SSB/SC signal is termed as Phase Shift Method and employs two balanced modulators as shown in Fig. 3.14. The balanced modulator 1 is given the modulating and carrier signals in the usual manner. The balanced modulator 2 is given these signals after a phase shift of 90°. If the carrier and modulating signals are assumed to be $E_c \sin \omega_c t$ and $E_m \sin \omega_m t$, then the output of the balanced modulator 1 is same as given by Eq. 3.14.

i.e.,
$$e_1 = K \frac{E_m . E_c}{2} [\cos (\omega_c - \omega_m) t - \cos (\omega_c + \omega_m) t] \quad \text{... (3.16)}$$

but input to balanced modulator 2 is $E_c \cos \omega_c t$ and $E_m \cos \omega_m t$. Therefore, the output of this circuit is given by the product term of these two signals.

$$\therefore\ e_1 = \frac{K E_c . E_m}{2} [\cos (\omega_c - \omega_m) t + \cos (\omega_c + \omega_m) t] \quad \text{... (3.17)}$$

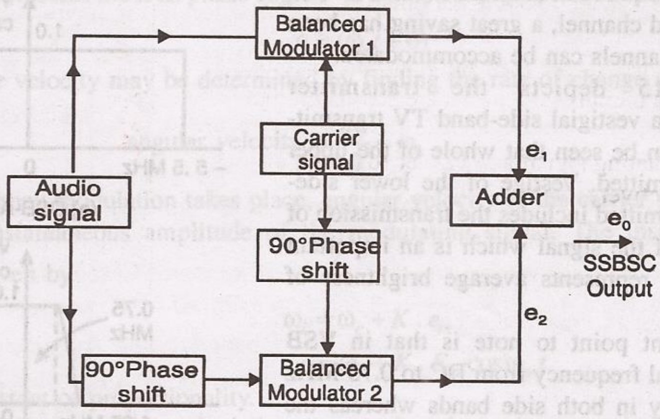


Fig. 3.14. Phase shift method of generating SSB/SC signals.

When these two voltages are added in the adder circuit, the upper side band vanishes leaving aside the low side-band alone.

Example 3.14. *An AM radio transmitter gives a power output of 5 kW when modulated to a depth of 95%. If after modulation by a speech signal which produces an average modulation depth of 20%, the carrier and one side band are suppressed, determine the average power in remaining output.*

Solution. Carrier power in case of 95% modulation depth is determined by using the relation

$$P_T = P_C\left(1+\frac{m_a^2}{2}\right)$$

where $$P_T = 5 \text{ kW and } m_a = 0.95.$$

$$P_C = \frac{P_T}{\left(1+\frac{m_a^2}{2}\right)} = \frac{5000}{1+\frac{(0.95)^2}{2}}$$

$$= \frac{5000}{1.451} = 3450 \text{ Watts}$$

With 20% modulation, the power in side bands is given by

$$P_{SB} = \frac{m_a^2 . P_c}{2} = \frac{0.2\times0.2\times3450}{2} = 69 \text{ Watts}$$

With SSB/SC transmission, carrier and one of the side bands are suppressed. The power output is, therefore, given by

$$P_{OUT} = \frac{P_{SB}}{2} = \frac{69}{2} \qquad \textbf{Ans. } P_{OUT} = 34.5 \text{ W}$$

3.2.6. Vestigial Side-band Systems (VSB). Picture signals of television occupy a bandwidth of about 6 MHz. If the transmission is done using the normal AM system, a bandwidth of about 12 MHz is required. In order to economise on bandwidth and still have the advantage of AM, (simple receiver design), one of the side-bands is completely transmitted while for the other side band only a *vestige* is transmitted. This results in considerably saving in the bandwidth. The overall bandwidth required in this case is about 7 MHz including the sound channel, a great saving has been made and more channels can be accommodated.

Figure 3.15 depicts the transmitter characteristicts of a vestigial side-band TV transmitting system. As can be seen that whole of the upper side-band is transmitted. Vestige of the lower side-band that are transmitted includes the transmission of DC components of the signal which is an important picture detail and represents average brightness of the picture.

An important point to note is that in VSB transmission, signal frequency from DC to 0.75 MHz are present equally in both side bands whereas the remaining signal frequencies are present only in the upper side band. If these frequencies are amplified equally by the receiver stages, then frequencies from 0 to 0.75 MHz will have a magnitude twice that of the remaining frequencies of the signal.

In order to avoid this, the response of a TV receiver is adjusted in such a manner that amplification at these frequencies is reduced to half of the amplification at the other frequencies, so that the output conforms to the original shape.

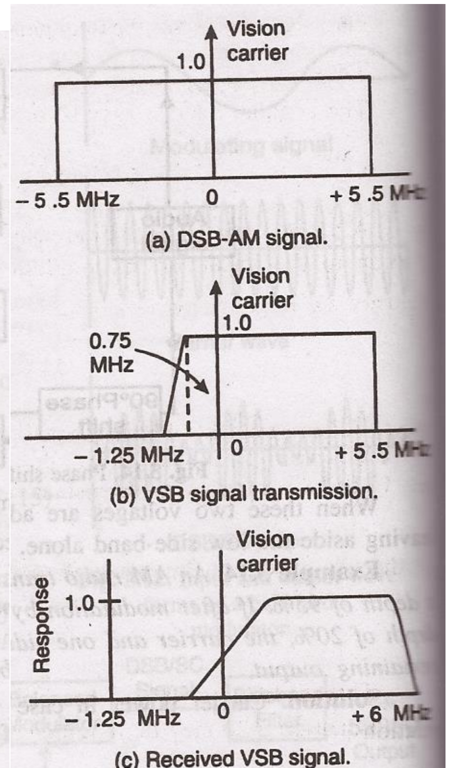


Fig. 3.15. VSB signal of a TV system

3.3. Frequency Modulation (FM)

Frequency modulation is the process of varying the frequency of a carrier wave in proportion to the instantaneous amplitude of the modulating signal without any variation in the amplitude of the carrier wave. Because the amplitude of the wave remains unchanged, the power associated with an FM wave is constant. Figure 3.16 depicts an FM wave.

As can be seen from the figure, when the modulating signal is zero, the output frequency equals f_c (centre frequency). When the modulating signal reaches its positive peak, the frequency of the modulated signal is maximum and equals $(f_c + f_m)$. At negative peaks of the modulating signal, the frequency of the FM wave becomes minimum and equal to $(f_c - f_m)$. Thus, the process of frequency modulation makes the frequency of the FM wave to deviate from its centre frequency (f_c) by an amount $(\pm \Delta f)$ where Δf is termed as the *frequency deviation* of the system. During this process, the total power in the wave does not change but a part of the carrier power is transferred to the side-bands.

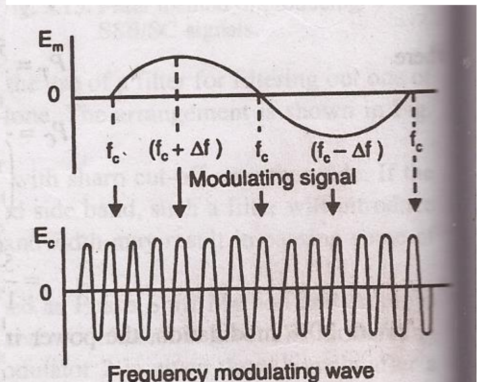


Fig. 3.16. Weve forms in a FM wave.

Assume the modulating signal to be represented by

$$e_m = E_m \cos \omega_m t$$

and the carrier wave being represented by

$$e_c = E_c \sin (\omega_c t + \theta) \quad \text{... (3.18)}$$

$(\omega_c t + \theta)$ represents the total phase angle F at a time t and q represents the initial phase angle.

Thus $$\phi = (\omega_c t + \theta)$$

The angular velocity may be determined by finding the rate of change of this phase angle.

i.e., $$\text{angular velocity} = \frac{d\phi}{dt} = \omega_c \qquad \text{... (3.19)}$$

After frequency modulation takes place, angular velocity of the carrier wave varies in proportion to the instantaneous amplitude of the modulating signal. The instantaneous angular velocity ω_i is given by

$$\omega_i = \omega_c + K \cdot e_m$$
$$= \omega_c + K \cdot E_m \cos\omega_m t \qquad \text{... (3.20)}$$

where K is a constant of proportionality.

Maximum frequency shift or deviation occurs when the cosine terms in Eq. 3.20 has a value ±1. Under this condition, the instantaneous angular velocity is given by

$$\omega_i = \omega_c \pm K \cdot E_m$$

so that the maximum frequency deviation Δf is given by

$$\Delta f = \frac{K\, E_m}{2\pi}$$

This gives $$K\, E_m = 2\pi \Delta f$$

Equation 3.20 may be rewritten as

$$\omega_i = \omega_c + 2\pi\Delta f \cos\omega_m t \qquad \text{... (3.21)}$$

Integration of Eq. 3.21 gives the instantaneous phase angle of the frequency modulated wave.

$$\phi_i = \int \omega_i \cdot dt$$
$$= \int (\omega_c + 2\pi\Delta f \cos\omega_m t) \cdot dt$$
$$= \omega_c t + \frac{2\pi\Delta f}{\omega_m} \sin\omega_m t + \theta_1$$

where θ_1 is a constant of integration representing a constant phase angle and may be neglected in the following analysis.

The instantaneous amplitude of the modulated waves is given by

$$e_{mod} = E_c \sin\phi_i$$
$$= E_c \cdot \sin\left(\omega_c + \frac{\Delta f}{f_m} \sin\omega_m t\right) \qquad \text{... (3.22)}$$

The ratio $\frac{\Delta f}{f_m}$ is termed as the modulation index of the frequency modulated wave and is denoted by m_f. It should be noted that for a given frequency deviation Δf, the modulation index varies with the modulating frequency f_m. A comparison of the modulation index m_a for the AM and for the frequency modulated wave shows that while m_a is given as the ratio of the change in the carrier amplitude due to amplitude modulation to the carrier amplitude, whereas m_f, modulation index for FM is given as the ratio of frequency diviation to the modulating frequency *i.e.*

$$m_f = \frac{\text{frequency deviation}}{\text{modulating frequency}} = \frac{\Delta f}{f_m} \quad \text{... (3.23)}$$

Substituting eq. 3.23 into eq. 3.22, the equation for FM is given as

$$e_{mod} = E_c \sin(\omega_c t + m_f \sin \omega_m t) \quad \text{... (3.24)}$$

It is seen that the modulating frequency decreases as the modulation index increases and the modulating voltage amplitude (*i.e.*, frequency deviation, Δf) remains constant. The modulation index for FM is the ratio of two frequencies and is measured in radians.

Example 3.15. *An FM wave is represented by the voltage equation* $e_{\text{mod}} = 10\sin(8\times10^6 t + 6\sin 3\times10^4 t)$ *calculate (a) the modulating frequency (b) the carrier frequency (c) the modulating index (d) the frequency deviation (e) and power dissipated in an 8 ohm load.*

Solution. (*a*) The modulating frequency $f_m = \dfrac{\omega_m}{2\pi}$

Given $\omega_m = 3\times10^4$ then

$$f_m = \frac{3\times10^4}{2\pi} = 4.77 \text{ KH}_z$$

(*b*) The carrier frequency $f_c = \dfrac{\omega_c}{2\pi} = \dfrac{8\times10^6}{2\pi} = 1.27 \text{ MH}_z$

(*c*) The modulating index $m_f = 6$ (from given voltage equation)

(*d*) The frequency deviation $\Delta f = m_f \times f_m = 6\times4.77 = 28.62 \text{ KH}_z$

(*e*) Power dissipated $P_d = \dfrac{(E_{rms})^2}{R} = \left(\dfrac{10\sqrt{2}}{8}\right)^2 = \dfrac{50}{8} = 6.25$ Watts

3.3.1. Frequency Spectrum of an FM wave. Different frequency components in an FM wave can be determined in the same wave as followed for AM wave *i.e.*, by expanding the expression for the waves. However, the analysis is rather complicated in this case and involves the use of Bessel functions.

$$\begin{aligned} e_{mod} &= E_c \sin(\omega_c t + m_f \sin \omega_m t) \\ &= E_c\{\sin\omega_c t \, . \cos(m_f \sin \omega_m t) + \cos\omega_c t . \sin(m_f \sin \omega_m t)\} \\ &= E_c[J_o(m_f)\sin\omega_c t + J_1(m_f)\{\sin(\omega_c + \omega_m)t - \sin(\omega_c - \omega_m)t\} \\ &\quad + J_2(m_f)\{\sin\omega_c + 2\omega_m)t + \sin(\omega_c - 2\omega_m)t\} \\ &\quad + J_3(m_f)\{\sin(\omega_c + 3\omega_m)t + \sin(\omega_c - 3\omega_m)t\} \\ &\quad + J_4(m_f)\{\sin(\omega_c + 4\omega_m)t + \sin(\omega_c - 4\omega_m)t\} \\ &\quad + ...] \end{aligned}$$

where J_0, J_1, J_2 ... are the coefficients of zero, first, second, order for the Bessel function and (m_f) is the argument. Values of $J_n(m_f)$ may be found out from the Bessel function chart given in Appendix A. A typical plot of the Bessel function is also given in Fig. 3.17.

In order to evaluate the amplitude of any side band, it is only necessary to find out the corresponding value of $J_n(m_f)$ and multiply it with E_c. For example, an FM wave with a maximum deviation $\Delta f = \pm 75$ KHz and a maximum audio frequency of 15 KHz has a modulation index, $m_f = 5$. The wave has a total of 8 upper side bands and an equal number of lower side bands. The magnitude of the 8th side band is only 2% of the carrier amplitude.

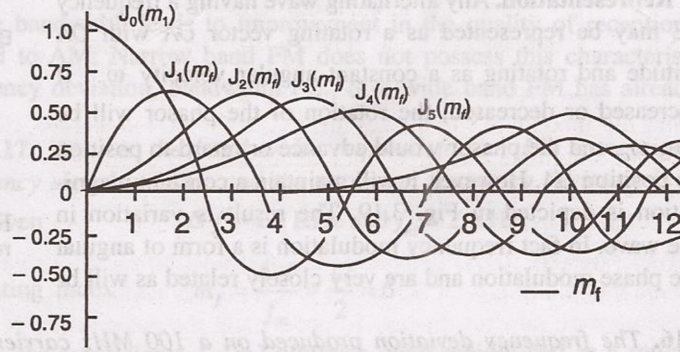


Fig. 3.17. a plot of Bessel function.

The FM wave as a matter of fact, contains an infinite number of side bands, each side band is separated from the next by f_m. However, out of these, there are only a few side bands which carry significant amount of power. The remaining side bands have such a low power that they get attenuated during propagation and do not convey any message to the receiver. It is the usual practice to notify all the side bands having amplitudes greater than 5% of the carrier as SIGNIFICANT SIDE BANDS and neglect all the remaining side bands with amplitude less than 5% of the carrier.

An increase in the modulating frequency signal amplitude at the transmitter results in a larger frequency deviation and as a consequence in a larger bandwidth of the modulated signal. This, if unchecked may sometimes result in overlapping of the upper side band components of one FM channel with the lower side band components of the adjacent FM channel. To avoid this possibility, the following limits have been set by FCC and CCIR.

1. Maximum permitted frequency deviation $= \pm 75$ KHz.
2. Frequency stability of the carrier $= \pm 2$ KHz.
3. Maximum allowed audio frequency = 15 KHz.
4. Guard bands = 50 KHz.
5. Maximum bandwidth allowed/channel = 200 KHz.

Frequency spectrum of an FM wave may be plotted in the usual way. Figure 3.18 shows the plot of a frequency modulated wave for $m_f = 0.5$ and $m_f = 5$. It can be seen from Fig. 3.17 as well as the Bessel function chart that in the plot for $m_f < 1.0$, there are a few side bands of large magnitudes. As m_f becomes large, the number of side bands frequencies increase but their amplitude becomes relatively small.

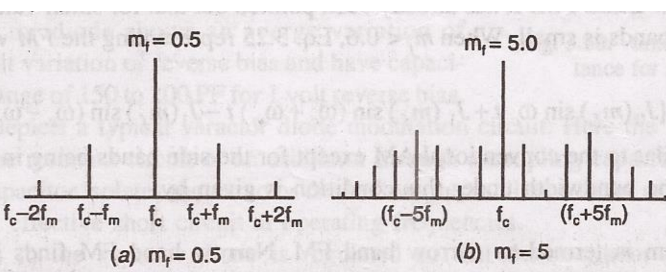


Fig. 3.18. Frequency spectrum of FM wave width (*a*) *mf* = 0.5 and (*b*) *mf* = 5.

Maximum frequency deviation and maximum audio frequency being fixed at $\pm$ 75 KHz and 15 KHz respectively, their ratio is termed as the *Deviation Ratio* (δ).

$$\therefore \quad \text{Deviation Ratio } (\delta) = \frac{\Delta f_{(max)}}{f_{m(max)}} = \frac{75}{15} = 5.$$

3.3.2. Vector Representation. Any alternating wave having a frequency f_c and amplitude E may be represented as a rotating vector OA with OA representing magnitude and rotating as a constant angular velocity ω_c. If its frequency is increased or decreased, the rotation of the phasor will be faster or slower than ω_c and the phasor would advance or retard to position OB or OC from the position OA. However, it will maintain a constant magnitude. Such a situation is depicted in Fig. 3.19. The result is variation in phase angle ϕ of the wave. In fact frequency modulation is a form of angular modulation just like phase modulation and are very closely related as will be seen later on.

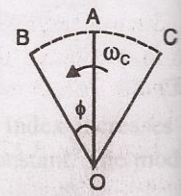


Fig. 3.19. Vector representation of an FM wave.

Example 3.16. *The frequency deviation produced on a 100 MHz carrier by a 5000 Hz signal is 50 KHz. Determine the angle of phase advance and retardation produced by this signal as also frequency deviation that would be produced by a signal of equal amplitude out of frequency 100 Hz.*

Solution. Instantaneous phase of FM wave is given as

$$\phi_i = \left(\omega_c t + \frac{\Delta f}{f_m} \sin \omega_m t\right) \quad \text{... (3.25)}$$

$$= (\omega_c t + m_f \sin \omega_m t)$$

Maximum advance or retard in the phase is given by

Advance or retard $= \Delta\phi_i = m_f$

When $\quad f_m = 5000$ Hz and $\Delta f = 50$ KHz

Phase advance/retard $= \dfrac{50000}{5000} = 10$ radians

For $\quad f_m = 100$ Hz and $\Delta f = 50$ KHz

Phase advance/retard $= \dfrac{50000}{100} = 500$ radians

Ans. $\Delta\phi_1 = 10$ rad

$\Delta\phi_2 = 500$ rad

3.2.3. Narrow Band FM. It has already been pointed out that for small valued of m_f, number of significant side bands is small. When $m_f < 0.6$, Eq. 3.25 representing the *FM* wave simplifies to become

$$e_{\text{mod}} = E_c\{J_0(m_f) \sin \omega_c t + J_1(m_f) \sin(\omega_c + \omega_m) t - J_1(m_f) \sin(\omega_c - \omega_m) t\} \quad \text{... (3.26)}$$

which is similar to the conventional AM except for the side bands being in phase quadrature with the carrier. The bandwidth under this condition is given by

$$BW = 2 f_m \quad \text{... (3.27)}$$

Such a system is termed as narrow band FM. Narrow band FM finds its use in Police, defence, fire services etc., with frequency deviation lying in the range of 15—25 KHz.

3.2.4. Wide Band FM. For values of modulation Index $m_f < 0.6$, the side bands produced cover a wide frequency spectrum but their amplitudes decrease. In such cases, the number of significant side bands increases and the system is termed as wide band FM.

Wide band FM requires a considerably larger bandwidth as compared to the corresponding AM wave. When $m_f > 5$, the coefficients $J_0 (m_f)$ diminish quite rapidly and the system bandwidth in a rough approximation may be given by

$$BW = 2 \, . \, m_f . f_m \qquad \text{... (3.28)}$$

Such a large bandwidth leads to improvement in the quality of reception, with very low noise as compared to AM. Narrow band FM does not possess this characteristic due to small bandwidth. Frequency deviation, bandwidth etc., of a wide band FM has already been given in sec. 3.3.

Example 3.17. *What is the band width required for an FM wave signal in which the modulating frequency signal is 2KHz and the maximum frequency deviation is 12 KHz ?*

Solution. Given $\Delta f = 12$ KHz and $f_m = 2$ KHz

The modulating index $m_f = \dfrac{\Delta f}{f_m} = \dfrac{12}{2} = 6$

Here $m_f > 5$, the coefficients $J_0 (m_f)$ diminish quite rapidly and as a rough approximation, $BW = 2\, m_f . f_m$.

Also the Carson's rule states approximately the bandwidth required to pass an *FM* is twice the sum of the deviation and the highest modulating frequency.

$$\begin{aligned} BW &= 2 \, . \, m_f . f_m \\ &= 2 \times 6 \times 2 \\ &= 24 \text{ KHz} \end{aligned}$$

Ans. $BW = 24$ KHz

3.2.5. F.M. Circuits. The frequency of an L—C oscillator is determined by tuning capacitor and the inductor. If the system includes a reactive element whose reactance can be varied by the modulating signal, this results in production of a frequency—modulated wave. This forms the basis of varactor diode modulator and reactance tube modulator circuits.

Alternately, an FM wave may be obtained by variation of phase of the carrier in proportion to the modulating signal as in phase modulation and will be considered along with phase modulation (P.M.).

Varactor Diode Modulation

Varactor diode is a specially fabricated PN Junction diode which is used as a variable capacitor in the reverse biased condition. This capacitor is dependent upon the magnitude of the reverse bias as shown in Fig. 3.20 and its capacitance is given by

$$C \propto \frac{1}{\sqrt{V}} \qquad \text{... (3.29)}$$

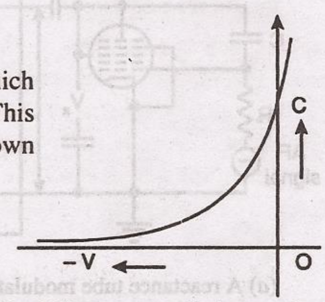


Fig. 3.20. Varactor diode capacitance for reverse bias

Silicon varactor diode shows an averge variation of (10—15) PF per volt variation of reverse bias and have capacitance, lying in the range of 150 to 200 PF for 1 volt reverse bias.

Figure 3.21 depicts a typical varactor diode modulation circuit. Here the varactor diode is connected across the resonant circuit of an oscillator through a coupling capacitor of relatively a large value. This capacitor isolates the varactor diode from the oscillator as far as DC is concerned while providing an effective short circuit at operating frequencies.

The DC bias to the varactor diode is regulated so that the oscillator frequency is not affected by varactor supply fluctuations. The modulating signal is fed in series with this regulated supply and at any instant, the effective bias to the varactor diode equals the algebraic sum of the DC bias voltage V and the instantaneous value of the modulating signal. As a result, the varactor capacitance varies with the modulating signal resulting in frequency modulation of the oscillator output.

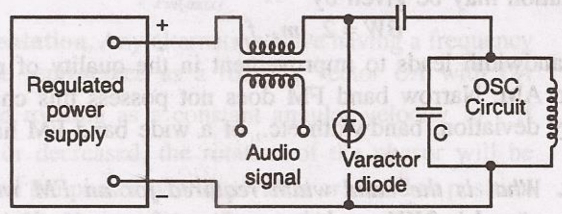
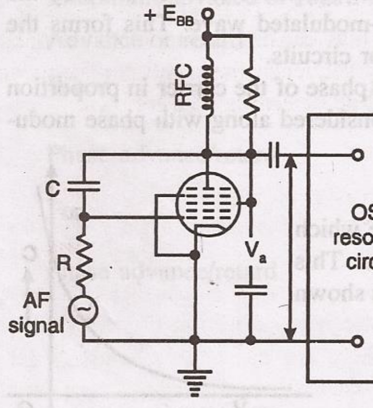


Fig. 3.21. A typical varactor diode modulator.

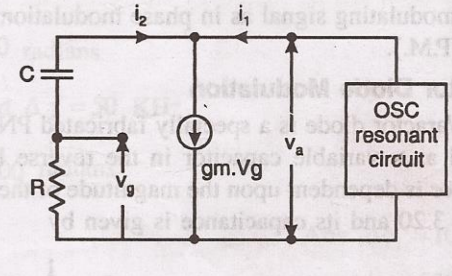
Reactance Tube Modulator

Reactance tube modulator is another circuit that was very widely employed for FM generation before the invention of varactor diodes. To understand the basic principle of reactance tube modulator, one must remember that in any tube, the anode current and voltage are in phase (neglecting plate load). If they could be made to have a phase difference of 90°, the tube will behave like a reactance.

Consider the circuit of a Fig. 3.22 (*a*). The circuit consists of a pentode tube with a phase shifting network C—R connected between anode and cathode as shown and the junction of this network is connected to the control grid. The tube is coupled across the oscillator resonant circuit. Figure 3.22 (*b*) shows the equivalent circuit of the modulator in constant current generator form where $g_m e_g$ is the current output of the equivalent generator. This output must, therefore, be equal to the sum of i_1 and i_2.



(*a*) A reactance tube modulator.



(*a*) AC equivalent circuit of a reactance tube modulator

Fig. 3.22

$$\therefore \qquad g_m \cdot e_g = i_1 + i_2 \qquad \text{... (3.30)}$$

but

$$i_2 = \frac{V_a}{R + \dfrac{1}{j\omega C}} = \frac{jV_a\omega C}{j\omega CR + 1}$$

and

$$v_g = i_2 R = \frac{jV_a \cdot R\omega C}{j\omega CR + 1} \qquad \text{... (3.31)}$$

Substitution of Eq. 3.31 in 3.30 yields

$$\frac{jg_m V_a . R.\omega C}{j\omega RC + 1} = i_1 + \frac{jV_a \omega C}{j\omega CR + 1}$$

or

$$i_1 = \frac{V_a(jg_m R\omega C - j\omega C)}{j\omega CR + 1}$$

Thus,

$$\frac{i_1}{V_a} = \frac{j\omega C(g_m R - 1)}{j\omega CR + 1} = \frac{(g_m R - 1)\,\omega C}{\omega CR - J}$$

If

$$g_m . R >> 1, \text{ then } \frac{i_1}{V_a} \approx \frac{\omega C g_m R}{\omega CR - j}$$

The values of C, R are so arranged that $i_2 << i_1$, so that i_1 may be taken as equal to the tube current i_a. In that case

$$\frac{i_1}{V_a} = \frac{i_a}{V_a} = Y_a = \frac{g_m\,\omega CR}{\omega CR - j}$$

Rationalising the equation given $Y_a = g_m \omega^2 C^2 R^2 + j\omega R g_m C,$ if $R << \frac{1}{\omega C}$. Therefore, the

$1 + (wCR)^2$

reactance tube behaves like a high resistance $\frac{1}{g_m \omega^2 C^2 R^2}$ shunted by a capacitance $g_m . R.C$ connected across the resonant circuit. This shunting high value resistance may be neglected and the modulation circuit can be assumed to work like a capacitor having value $g_m\ C.R$ connected across the resonant circuit.

The application of modulating signal changes the grid bias causing the tube g_m to change. This results in the change effective capacitance and as a consequence the oscillator frequency. The corresponding frequency change for a change in g_m may be determined as follows.

Let

$$f_0 = \frac{1}{2\pi\sqrt{L\,C_T}}$$

and

$$f_0 - \Delta f = \frac{1}{2\pi\sqrt{L\,(C_T + \Delta C)}}$$

$$= \frac{1}{2\pi\sqrt{LC_T\left(1 + \frac{\Delta C}{C_T}\right)}}$$

$\therefore$

$$\Delta f = \frac{1}{2\pi\sqrt{L\,.\,C_T}}\left[1 - \frac{1}{\left(1 + \frac{\Delta C}{C_T}\right)^{1/2}}\right]$$

$$= f_0\left[1 - \left(1 + \frac{\Delta C}{C_T}\right)^{-1/2}\right]$$

or $$\Delta f / f_0 = 1-(1+\Delta C/C_T)^{-1/2}$$
$$= 1-1+\Delta C/2C_T + ...\qquad ...(3.32)$$

Hence $$\Delta f / f_0 \approx \Delta C/2C_T$$

Now $$\Delta C = \Delta g_m R$$

As CR is constant, ΔC varies as Δg_m. Thus for linear frequency modulation ΔC or Δg_m must be varied linearly with the modulating signal.

Transistor Reactance Modulator

Transistor reactance modulator is also widely employed for FM generation. Fig 3.23 shows a transistor reactance modulator, transistor T_1 circuit is a RC capacitive transistor reactance modulator, which is connected to the tank circuit of the LC oscillator.

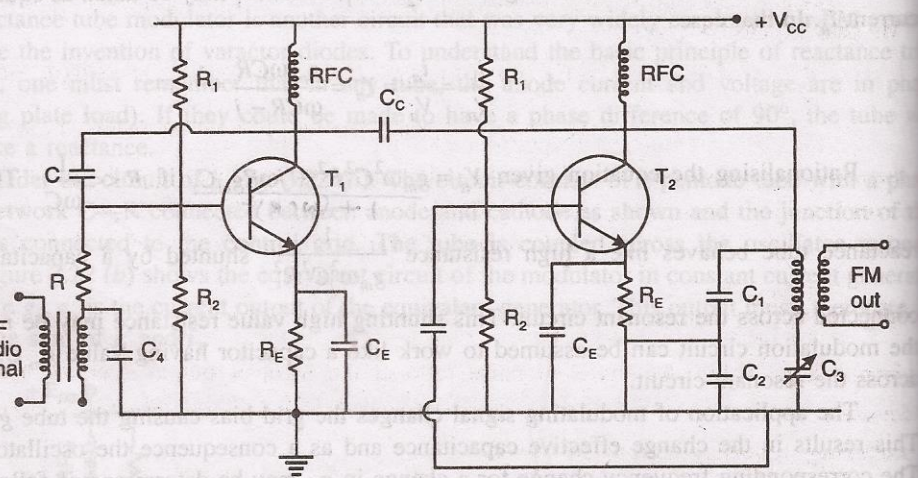


Fig. 3.23. A Transistor reactance modulator

AF signal is applied to the base of transistor T_1 which changes the capacitive reactance of the output which is connected to the L-C tank circuit of oscillator. Transistor T_2 is a clapp-oscillator circuit. FM output is obtained from the clapp-oscillator tank circuit. Instead of bipolar transistor — a FET can also be used as a reactance modulator. An FET basic reactance modulator circuit is shown in Fig. 3.24. The transconductance g_m of the FET changes according to the modulating signal, which is applied at the gate and source of the FET. The oscillator tank circuit is connected at the drain and source. The FM output is proportioanl to the transconductance of the FET.

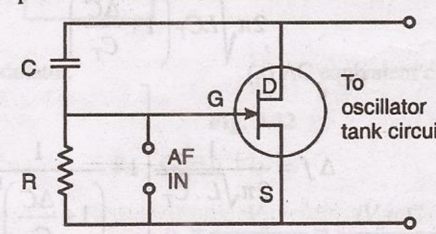


Fig. 3.24. A basic FET reactance modulator.

3.4. Interference in AM and FM Systems

Interference in AM and FM reception may be caused either by a strong distant station working on the same channel or by a station working on a channel adjacent in frequency to the desired channel to which a receiver is tuned. In this section, the effect of these interfering signals on different modulation systems is discussed.

Consider at first an interfering station working on the same channel. As this station has the carrier frequency very close to the desired signal, the two signals add vectorially, causing amplitude as well as phase modulation, as shown in Fig. 3.28. In general, consider an interfering signal with a frequency f_i that is close to the carrier frequency f_0 of the signal. As a result, the interfering signal rotates at an angular velocity $2\pi(f_i - f_0)$ relative to the desired signal, causing amplitude and phase change of the latter.

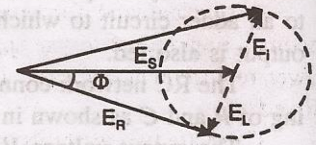


Fig. 3.28. Production of amplitude and phase shift by an interference signal

In AM receivers, the phase shift of the carrier is not of much importance, since it is the resulting AM that causes serious problems. If the two signals have the same frequency, $(f_i - f_0)$ becomes zero and these signals add algebraically. These signals are simultaneously present at the output, making reception unpleasant and unintelligible. The degree of such interference depends upon the relative amplitude of the interference signal. If the interference signal frequency is different than the signal frequency the two beat together, thereby producing audible beat note at the receiver output.

The FM receivers are fitted with limiter stages, that make these receivers immune to amplitude variations in the desired signal due to interfering signals. It is the change in the phase of the desired signal because of the interference signal that must be considered in FM reception.

The maximum phase deviation $\pm\Delta\theta$ caused by interfering signals is given by

$$\Delta\theta = \sin^{-1}\frac{E_i}{E_s}.$$

The corresponding frequency deviation caused by this phase change is given by

$$\Delta f = \Delta\theta .. f_m$$

where f_m stands for frequency difference between the desired and interference signals. Even if this difference is assumed to be 15 KHz and the amplitude of the interfering signal is assumed to be as large as signal voltage, the phase difference ϕ produced is only 45° or $\pi/4$ radians and the frequency deviation so produced equals 15 KHz $\times\frac{\pi}{4}$ = 11.5 KHz. The frequency deviation so produced is only about one-seventh of the deviation of desired signal. Thus, this signal can cause little interference in the receiver output. If the frequency difference is assumed to be more than 15 KHz, a larger frequency deviation is produced but this frequency being outside the audio range, is not reproduced at the receiver output.

Lastly, if the desired and interfering signal are at the same frequency, the frequency deviation produced is zero. In this case, the FM receiver will receive the stronger of the two signals. In other words, the FM receiver captures the stronger of the two signals. This is termed as *capture effect*. This interference is also reduced by the use of VHF carrier frequencies which can only be received by line of sight propagation and also the separation of FM stations by guard bands of about 50 KHz that help to reduce this effect.

Example 3.19. *In an FM wave, the frequency deviation is 25 KHz. What maximum phase deviation does this represent if the modulation signal is (a) 100 Hz and (b) 1000 Hz ?*

Solution. $\Delta f = 25$ KHz

Now $\Delta \theta = m_f = \dfrac{\Delta f}{f_m}$

(*a*) When $f_m = 100$ Hz

$$\Delta \theta = m_f = \frac{25000}{100} = 250 \text{ radians}$$

(*b*) When $f_m = 10000$ Hz

$$\Delta \theta = m_f = \frac{25000}{10000} = 2.5 \text{ radians}$$

Ans. (*a*) = 250 radians, (*b*) = 2.5 radians.

Example 3.20. *Explain reactance tube modulater for generating FM. A pentode having g_m variation by 0.1 mA/V for 1 volt change in grid bias is used as a reactance tube. A capacitor of 0.5 PF is connected between anode and grid and a resistance of 1 K between grid and cathode. The oscillator operates at 10 MHz with a tank circuit capacitance of 100 PF. If an AF voltage of 1.414 volts (RMS) is applied to the grid, calculate maximum frequency deviation.*

Solution. $g_m = 0.1$ mA/V

$C = 0.5$ PF

$R = 1$ K

$f_0 = 10$ MHz

AF voltage $= 1.414$ V RMS

$= 2.0$ V Peak

Tunning capacitor $C_t = 100$ PF

Now $\Delta C = g_m \cdot$ C.R. × (AF SIGNAL)

$= 0.1 \times 10^{-3} \times 0.5 \times 10^{-12} \times 10^{3} \times 2.0$

$= 0.1$ PF

Now $\Delta f / f_0 = \Delta C / 2C_t$

$$\therefore \quad \Delta f = f_0 \cdot \frac{\Delta C}{2C_t} = \frac{10 \times 10^{6} \times 0.1 \times 10^{-12}}{2 \times 100 \times 10^{-12}} = 5000 \text{ Hz}$$

Ans. $\Delta f = 5$ KHz

Example 3.21. (*a*) *Explain narrow band FM and wide band FM and discuss their bandwidth requirements in relation to FM. Discuss the relative performance of the above systems with regard to noise.*

(*b*) *A 100 MHz carrier is frequency modulated by a 10 KHz signal, so that the maximum frequency deviation is 1 MHz. Determine the approximate bandwidth of the system.*

Solution. For part (*a*) see text.

(*b*) $BW \approx 2\, m_f \cdot f_m$

Now $m_f = \Delta f / f_m$

$\therefore \quad BW \approx 2 \times \Delta f = 2 \times 1 \text{ MHz} = 2$ MHz **Ans.** 2 MHz

Example 3.22. (*a*) *What are the advantages of Frequency Modulation ? Show how a varactor can be used to produce a FM signal.*

(*b*) *The carrier frequency of a broadcast signal is 100 MHz. Maximum deviation is 75 KHz. If the highest audio frequency modulating the carrier is limited to 15 KHz, what is the approximate bandwidth required ?*

Solution. For part (*a*) see text.

(*b*) $$BW \approx 2 \,.\, m_f \,.\, f_m$$

Now $$m_f = \Delta f / f_m$$

$$BW = 2 .\, \Delta f = 2 \times 75 = 150 \text{ KHz.}$$ **Ans.** $BW = 150$ KHz

Example 3.23. (*a*) *Give the circuit of a balanced modulator for suppressed carrier AM signal transmission and explain how the circuit works.*

(*b*) *In Fig. 3.36, $v_c(t) = K \sin \omega_s t$ and $v_m = M \sin \omega_m t$. Show that for $m \le 1$, $v_0(t)$ approximates a phase modulated wave. Sketch the variation of instantaneous frequency of $v_0(t)$ with time.*

Solution. For part (*a*) see text.

(*b*) Now $v_c(t) = K \sin \omega_c t$

and $v_m(t) = M \sin \omega_m t$

Multiplier output $= v_c(t) \,.\, v_m(t)$

$$= MK \sin \omega_c t \,.\, \sin \omega_m t.$$

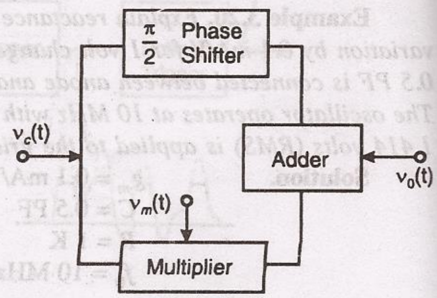


Fig. 3.36.

Phase shifter output $= K \sin\left(\omega_c t + \dfrac{\pi}{2}\right)$